1

KINEMATICS

Kinematics of machines is a study of relative motion between machine parts, without considering the forces which produce the motion. More particularly, kinematics is the study of position, displacement, velocity, and acceleration. The study of forces acting on the parts of a machine and the motion resulting from these forces is known as *dynamics of machines*.

Definitions

Element or link : Each part of a machine, which has motion relative to some other parts is termed an *element* or *link*. Generally, a link is a rigid member with provision at each end for connection to other links. A link need not necessarily be rigid body, but it must be resistant body, i.e., it should be of ample strength to transmit the required force with negligible deformation. Thus, incompressible fluid used in hydraulic press, jacks, brakes, hydraulic drives, etc., and belt, rope and chain are considered as links. The link which is stationary and which supports the moving members is called *frame*.

Links can be classified into singular (unitary), binary, ternary, quaternary, etc., depending upon its ends on which revolute joint can be placed for pairing with other link. Fig. 1.1 shows different types of links and there is no relative motion between the joints within the link.

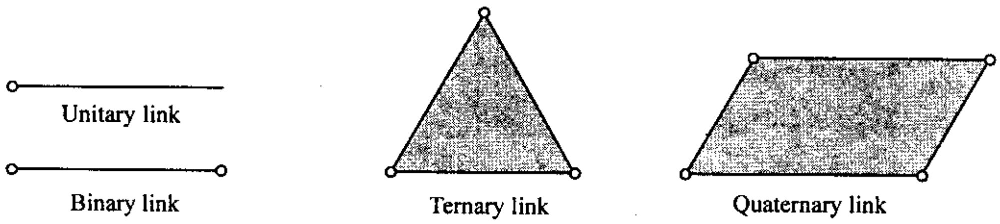


Fig. 1.1

Joint : A *joint* is a movable connection between links and allows relative motion between the links. The two primary joints are the *revolute* and *sliding joint*. The revolute joint is also called a *pin* or *hinge joint*. It allows pure rotation between the two links that it connects. The sliding joint is also called a *prism joint*. It allows linear sliding between the links that it connects. Fig. 1.2 illustrates these two primary joints.

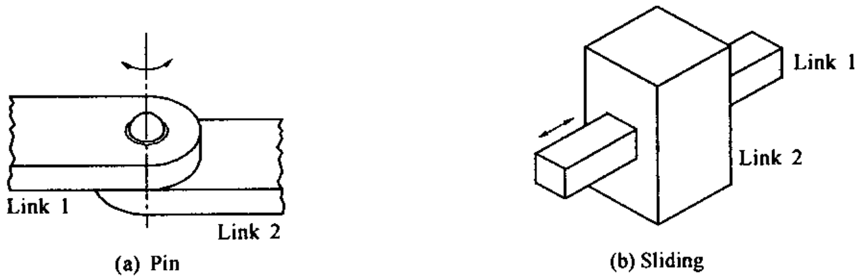


Fig. 1.2 Primary joints

Kinematics pair

When two elements or links are connected together in such a way that their relative motion is constrained, form a *kinematic pair*. Therefore, in order to compel a body to move in a definite path, it must be paired with another. If the constraint is not complete (not definite path) the pair is termed as *incomplete* or *unsuccessful*.

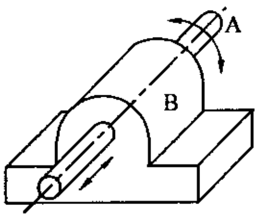


Fig. 1.3

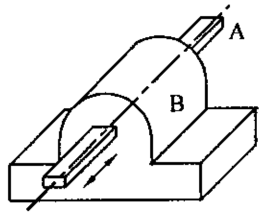


Fig. 1.4

Fig. 1.3 is an example of incomplete constraint. There are three possible motions between A and B. (1) Sliding, (2) Rotational, and (3) Partly sliding and partly rotational. In fig. 1.4a rectangular bar A can only slide relative to B and the constraint is therefore complete.

Kinematic pairs may be classified according to the following ways.

1. Type of contact between the elements
2. Type of relative motion between the elements.
3. Type of closure

Classification of pairs based on type of contact

According to the nature of contact, pairing of links can be classified into *higher paring*, and *lower pairing*. In higher pairing, the contact between the mating links is theoretically a point or line. Example of higher pair is that between balls and races of ball bearing, meshing gear teeth etc. If there is surface contact between the mating links, the pair thus obtained is termed as lower pair. Examples of lower pairs are: the journal bearing, slider, etc. All sliding pair, turning pair and screw pairs form lower pairs.

A higher pair will produce a higher contact stresses in the links. Lower pair joints are frequently used in mechanism design practice. They give good service because wear is spread out over the contact surface.

Classification of pairs based on type of relative motion

The nature of contact between the elements of a pair must be such as to permit the required relative motion. Various kinds of relative motion, commonly required are sliding, turning, screw motion, rolling, and spherical motion. Pairs, which permit these kinds of motion, are respectively called sliding pairs, turning pairs, screw pairs, rolling pairs, and spherical pairs.

Sliding pair: When two elements are so connected that one is constrained to have a sliding motion relative to the other, it forms a sliding pair as in fig. 1.5. *Example :* Cross head and its guide, surface of the piston and the cylinder wall, die-block and slotted lever.

Turning pair : When two elements are so connected that one is constrained to turn about a fixed axis of other, it forms a turning pair as shown in fig.1.6. *Example:* Crank shaft turning in a bearing, lever and its fulcrum, crank pin and piston pin.

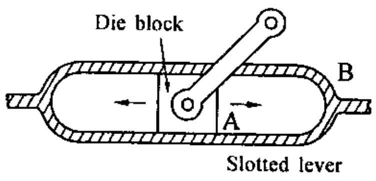


Fig. 1.5

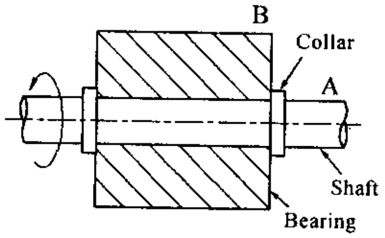


Fig. 1.6

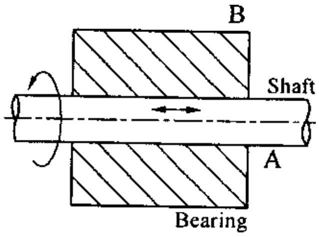


Fig. 1.7

Cylindrical pair: A cylindrical pair permits a relative motion which is a combination of rotation and translation. A shaft free to rotate in a bearing and also free to slide axially in the bearing is an example of cylindrical pair (fig. 1.7).

Screw pair: When one element turns about the other element by means of threads, it forms a screw or helical pair. The relative motion being the combination of sliding and turning as shown in fig. 1.8. *Example :* Bolt and nut, screw-jacks, screw presses and lead screw of a lathe.

Spherical pair : When one element is in the form of sphere turns about the other fixed element, it forms a spherical pair as shown in fig. 1.9. This joint permits relative rotation about three mutually perpendicular axes. *Example :* Ball and socket joint.

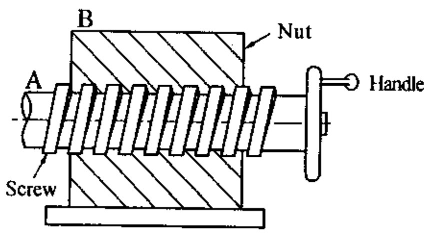


Fig. 1.8

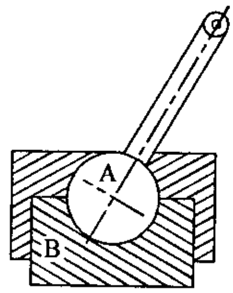


Fig. 1.9

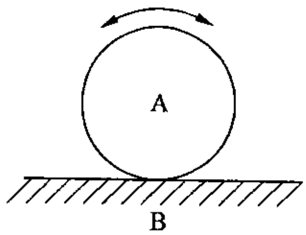


Fig. 1.10

Rolling pair: When two elements are so connected that one is constrained to roll in other fixed element, it forms a rolling pair as shown in fig. 1.10. *Example :* Ball and roller bearing.

Classification of pairs based on type of closure

Another way of classifying pairs is to group them as (i) *Closed kinematic pairs* (ii) *Open kinematic pairs*.

Closed kinematic pair: When two elements of a pair are held mechanically in such a manner that only required type of relative motion occurs is called a closed pairs. All lower pairs are closed pairs.

Open kinematic pair: When two elements are held in contact by the action of external forces are called open kinematic pair. *Example:* Cam and spring loaded follower, foot-step bearing, etc.

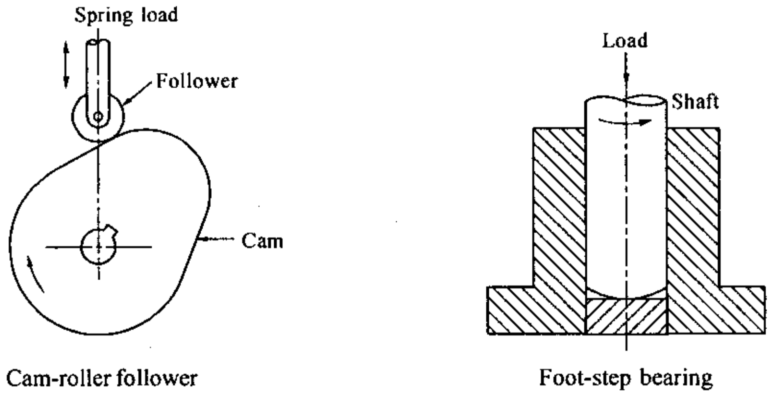


Fig. 1.11 Open kinematic pair

Kinematic chain

A *kinematic chain* is any assemblage of rigid links connected through pairs, permitting relative motion between links. A chain is called a *closed chain* when links are so connected in sequence that first link is connected to the last link. When the first link is not connected to the last link, the chain is called an *open chain*. *Example:* manipulator of the robot.

The relation between the number of links and the number of pairs may be expressed as,

$$n = 2P - 4$$

Where n = Number of links

P = Number of pairs

The relation between the number of links and the number of pairs may be expressed as,

$$n = \frac{2(j+2)}{3}$$

Where j = Number of joints

Mechanism

A mechanism is a constrained kinematic chain, with one link fixed. It is used to transmit or transform motion.

Inversion : A kinematic chain becomes a mechanism when one of its link is fixed. Utilizing alternate links to serve as the fixed link is termed *kinematic inversion.* As different links are chosen as a frame, the relative motion of the link is not altered, but changes their absolute motion relative to the frame. Thus as in fig. 1.12 any one of the links may be arbitrarily selected as the fixed link and each arrangement is an inversion of the others. The number of inversion is equal to the number of links in the kinematic chain.

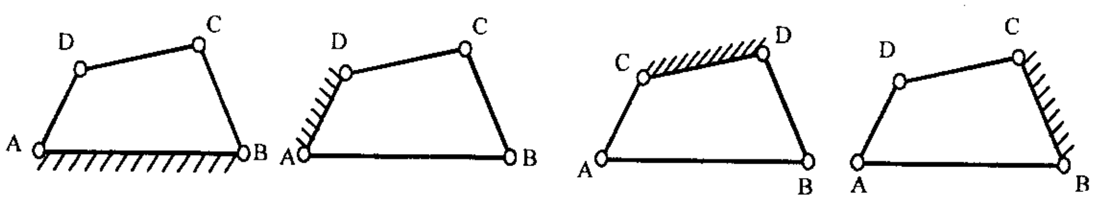


Fig. 1.12

Machine

A machine is a mechanism or group of mechanisms used to perform useful work. It is used to alter, transmit and direct forces to accomplish a specific work.

Difference between mechanism and machine

Mechanism	Machine
1. A mechanism is meant for transmit or transform motion.	A machine is meant for transmitting energy or to do useful work.
2. No mechanism is necessarily a machine.	A machine is series or train of mechanism.
3. A mechanism is a working model of any machine. *Example :* clock, mini-drafter, etc.	A machine is a practical development of any mechanism. *Example*: Steam engine, shaper, etc.

Difference between machine and structure

Machine	Structure
1. Machine modifies or transmit energy to do some kind or work.	Structure modifies and transmit forces only.
2. Relative motion exists between its member *Example :* Steam engine, shaper etc.	No relative motion exists between its member. *Example:* Roof truss.

Degrees of freedom

Degrees of freedom describes the number of ways a link of a mechanism can move. A free body in space has six degrees of freedom i.e., three translations and three rotations about the three mutually perpendicular axes. In forming kinematic pair, one or more constraints must be imposed.

∴ Degrees of freedom of a kinematic pair = 6 – Number of restraints

The degrees of freedom of various joints found in planar mechanisms are given in table. 1.

Table : 1

Joint	Nature of motion	Degrees of freedom
Revolute (hinge)	Pure rotation	1
Prismatic (slider)	Pure sliding	1
Cam or Gear	Rolling and sliding	2
Rolling contact (roller bearing)	Pure rolling	1

Note that a revolute joint connecting k links at a single point must be counted as $(k-1)$ joints.

Mobility

Mobility of a mechanism is the number of degrees of freedom it possesses. The degrees of freedom for planar mechanisms can be calculated through Gruebler's mobility equation:

$$M = 3(n-1) - 2j_1 - j_2$$

where M = Mobility or number of degrees of freedom

n = Total number of links including the frame in the mechanism

j_1 = Number of joints having one degree of freedom

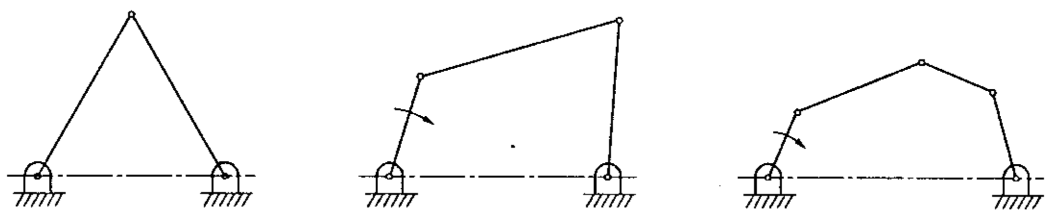
j_2 = Number of joints having two degrees of freedom

If $M > 0$, it gives a mechanism with M degrees of freedom

If $M = 0$, it gives a statically determinate structure

If $M < 0$, it gives a statically indeterminate structure

Mechanisms with one degrees of freedom are termed *constrained mechanisms*. Most mechanisms used in machines are constrained. In constrained mechanism, the movement of one link causes a definite predictable movement of the other links as shown in fig. 1.13(b).



(a) Locked mechanism ($m = 0$) (b) Constrained mechanism ($m = 1$) (c) Unconstrained mechanism ($m = 2$)

Fig. 1.13

Mechanisms with zero, or negative degrees of freedom are termed *locked mechanisms*. These mechanisms are unable to move and form a structure. It is the basis of structural trusses. A locked mechanisms is shown in fig. 1.13(a).

Mechanisms with more than one degrees of freedom are termed *unconstrained mechanisms*. In unconstrained mechanism the movement of one link does not cause a definite predictable movement of the other links as shown in fig. 1.13(c).

Example 1.1

Determine the mobility of the structure shown in fig. 1.14.

Solution :

Number of links $n = 3$, Number of revolute joints = 3

Number of one degree of freedom joints, $j_1 = 3$

Number of two degrees of freedom joints, $j_2 = 0$

$\therefore$ Mobility $M = 3(n-1) - 2j_1 - j_2$

$= 3(3-1) - 2 \times 3 - 0 = 0$

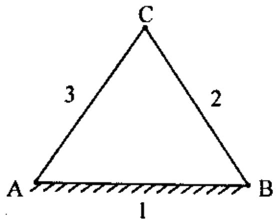


Fig. 1.14

Example 1.2

Determine the mobility of the four-bar linkage shown in fig. 1.15

Solution :

Number of links $n = 4$, Number of revolute joints = 4

Number of one degree of freedom joints $j_1 = 4$

Number of two degrees of freedom joints $j_2 = 0$

$\therefore$ Mobility $M = 3(n-1) - 2j_1 - j_2$

$= 3(4-1) - 2 \times 4 - 0 = 1$

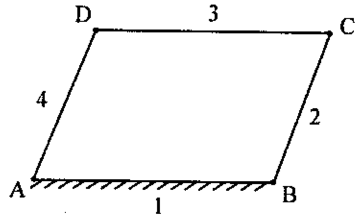


Fig. 1.15

Example 1.3

Determine the mobility of the slider crank mechanism shown in fig. 1.16

Solution :

Number of links $n = 4$ (including the frame)

Number of revolute joints = 3 and number of slider = 1

Total number of one degree of freedom joints $j_1 = 4$

Number of two degrees of freedom joints $j_2 = 0$

$\therefore$ Mobility $M = 3(n-1) - 2j_1 - j_2$

$= 3(4-1) - 2 \times 4 - 0 = 1$

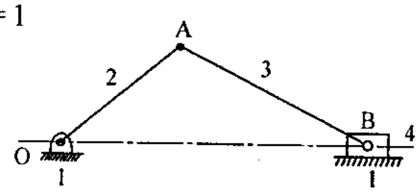


Fig. 1.16

Example 1.4

Determine the mobility of the mechanism shown in fig. 1.17

Solution :

Number of links $n = 4$ (including the frame) The revolute joint at O_1, O_2, and O_3 each one degreeof freedom. The revolute joint at A, connecting three links must be counted as $3 - 1 = 2$ revolute joints.

Total number of one degree of freedom joints $j_1 = 5$

Number of two degrees of freedom joints $j_2 = 0$

$\therefore$ Mobility $M = 3(n-1) - 2j_1 - j_2$

$= 3(4-1) - 2 \times 5 - 0 = -1$

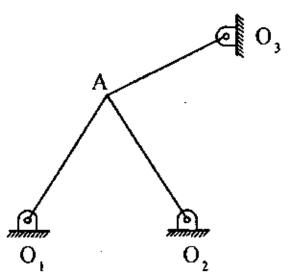


Fig. 1.17

Example 1.5

Determine the mobility of the mechanism shown in fig. 1.18

Solution :

Number of links $n = 4$

Number of revolute joints = 3

$\therefore$ Number of one degree of freedom joints $j_1 = 3$

The follower 3 will have rolling and sliding motion

$\therefore$ Number of two degrees of freedom joints $j_2 = 1$

Mobility $M = 3(n-1) - 2j_1 - j_2$

$= 3(4-1) - 2 \times 3 - 1 = 2$

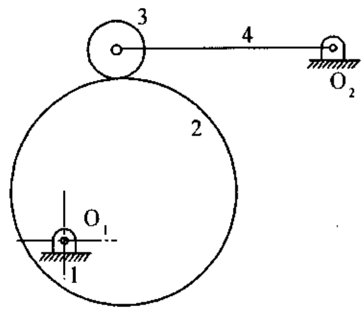


Fig. 1.18

Example 1.6

Find the degrees of freedom of the following mechanism shown in fig. 1.19

Solution : **(VTU, Aug. 2001)**

Number of links $n = 10$

The revolute joints at A and D have each one degree of freedom. Joints B, C, E and F connecting three links have each two revolute joints. Joint G connecting four links have three revolute joints.

Total number of one degree of freedom joints $j_1 = 13$

Number of two degrees of freedom joints $j_2 = 0$

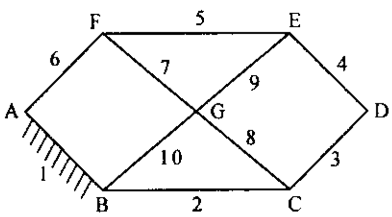


Fig. 1.19

$\therefore$ Degrees of freedom of the mechanism (mobility) $M = 3(n-1) - 2j_1 - j_2$

$= 3(10-1) - 2 \times 13 - 0 = 1$

Example 1.7

Determine the mobility of the device shown in fig. 1.20

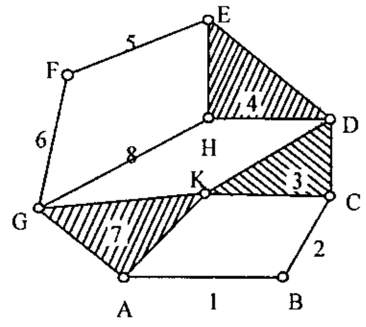


Fig. 1.20

Solution :

Number of links $n = 8$

The revolute joint at A, B, C, D, E, F, H and K have each one degree of freedom. The revolute joint at G connecting the three links namely 6, 7 and 8 has two revolute joints

Total number of one degree of freedom joints $j_1 = 10$

Number of two degrees of freedom joints $j_2 = 0$

$\therefore$ Mobility $M = 3(n-1) - 2j_1 - j_2$

$= 3(8-1) - 2 \times 10 - 0 = 1$

Example 1.8

Determine the mobility of the device shown in fig 1.21

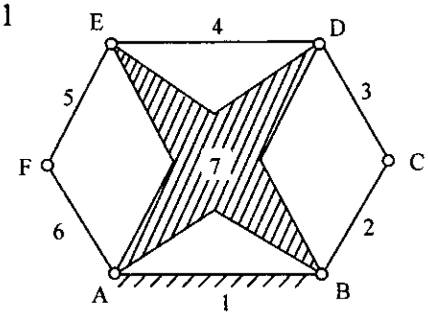


Fig. 1.21

Solution :

Number of links $n = 7$

The revolute joints at C and F have each one degree of freedom. The revolute joints at A, B, D, and E each connecting three links have two revolute joints.

$\therefore$ Total number of one degree of freedom joints

$j_1 = 10$

Number of two degrees of freedom joints $j_2 = 0$

Mobility $M = 3(n-1) - 2j_1 - j_2$

$= 3(7-1) - 2 \times 10 - 0 = -2$

Example 1.9

Determine the mobility of the linkage shown in fig. 1.22

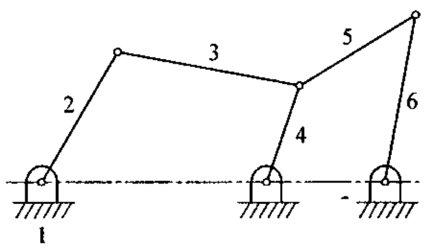


Fig. 1.22

Solution :

Number of links $n = 6$

Links 3, 4 and 5 are connected to the same revolute joint axis. The connectivity at that joint is $3 - 1 = 2$.

$\therefore$ Number of one degree of freedom joints

$j_1 = 5 + 2 = 7$

Number of two degrees of freedom joints $j_2 = 0$

Mobility $M = 3\,(n-1) - 2\,j_1 - j_2$

$= 3\,(6-1) - 2 \times 7 - 0 = 1$

Example 1.10

Determine the mobility of the linkage shown in fig. 1.23

Solution :

Number of links $n = 7$

Number of one degree of freedom joints $j_1 = 8$

(all joints have connectivity one)

Number of two degrees of freedom joints $j_2 = 0$

Mobility $M = 3\,(n-1) - 2\,j_1 - j_2$

$= 3\,(7-1) - 2 \times 8 - 0 = 2$

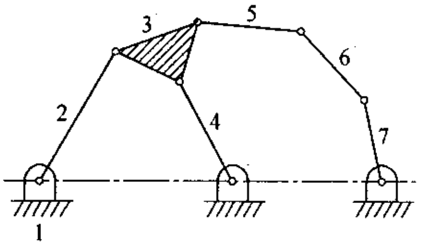


Fig. 1.23

Example 1.11

Determine the degree of freedom of the linkage shown in fig. 1.24. **(VTU, July 2005)**

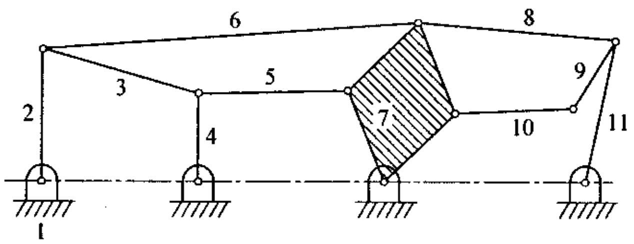


Fig. 1.24

Solution :

Number of links $n = 11$

There are 4 revolute points where 3 links are connected

i.e., links 2-3-6, 3-4-5, 6-7-8, and 8-9-11

Total number of one degree of freedom joints

$j_1 = 7 + 4\,(3-1) = 15$

Number of two degree of freedom joints $j_2 = 0$

Mobility $M = 3\,(n-1) - 2\,j_1 - j_2$

$= 3\,(11-1) - 2 \times 15 - 0 = 0$

Example 1.12

Determine the mobility of the mechanism shown in fig. 1.25 **(VTU, July 2004)**

Solution :

Number of links $n = 4$

Link 2 and 3 will have sliding motion only

$\therefore$ Number of one degree of freedom joints

$$j_1 = 2 \text{ pins} + 1 \text{ slide} = 3$$

Number of two degrees of freedom joints $j_2 = 0$

Mobility $M = 3(n-1) - 2j_1 - j_2$

$$= 3(3-1) - 2 \times 3 - 0 = 0$$

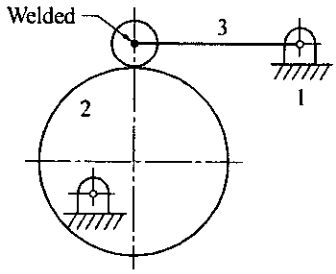


Fig. 1.25

Example 1.13

Calculate the mobility of the mechanism shown in fig. 1.26.

Solution :

Number of links $n = 6$

Links 3-4-5 are connected to one revolute joint.
The connectivity of this joint is $3 - 1 = 2$.

Number of single degree of freedom joints

$$j_1 = (4+2) \text{ pins} + 1 \text{ slide} = 7$$

Number of two degrees of freedom joints $j_2 = 0$

Mobility $M = 3(n-1) - 2j_1 - j_2$

$$= 3(6-1) - 2 \times 7 - 0 = 1$$

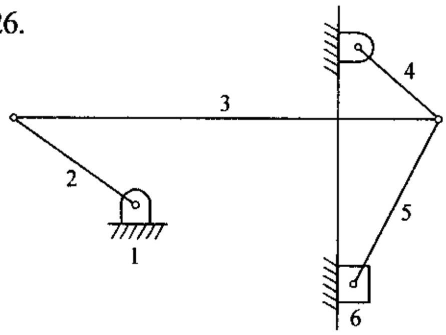


Fig. 1.26

Example 1.14

Calculate the mobility of the mechanism shown in fig. 1.27.

Solution :

Number of links $n = 5$

Number of single degree of freedom joints

$$j_1 = 4 \text{ pins} + 1 \text{ slide} = 5$$

The pin in a slot joint permits 2 degrees of freedom.

Number of two degrees of freedom joints $j_2 = 1$

Mobility $M = 3(n-1) - 2j_1 - j_2$

$$= 3(5-1) - 2 \times 5 - 1 = 1$$

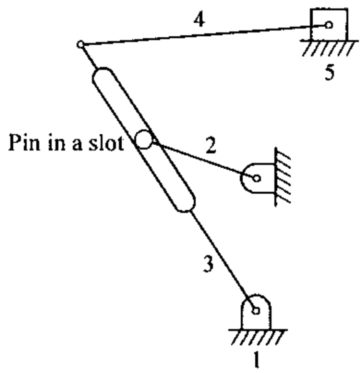


Fig. 1.27

Example 1.15

Determine the mobility of the device shown in fig. 1.28. (VTU, July 2004)

Solution :

Number of links $n = 3$

Number of single degree of freedom joints

$j_1 = 2$ revolute joints $= 2$

Number of two degrees of freedom joints $j_2 = 1$

(rolling and sliding between contact surface)

Mobility $M = 3(n-1) - 2j_1 - j_2$

$= 3(3-1) - 2 \times 2 - 1 = 1$

This is one degree of freedom mechanism

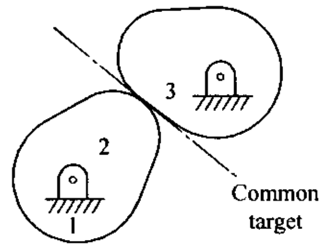


Fig. 1.28

Kinematic chains with three lower pairs

A kinematic chain with three lower pairs consists of either three sliding pairs (wedge) or the one, which consists of a turning, a sliding and a screw pair (screw press). It is impossible to have a kinematic chain, which consists of only three turning pairs.

Wedge : Wedge is a mechanical element for producing motion or exerting force. In the arrangement shown in fig. 1.29, the force P acting on the wedge A causes the slide C to move against a resistance W.

The three sliding pairs between the links are :

i) Wedge A and the frame B
ii) Wedge A and the slide C, and
iii) Slide C and the guide B.

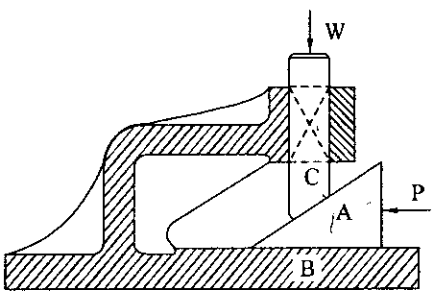


Fig. 1.29

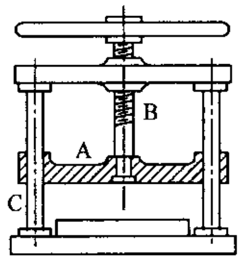


Fig. 1.30

Screw press : A simple hand press is shown in fig. 1.30

It consists of three lower pairs namely,

i) A sliding pair between the links A and C
ii) A turning pair between the links A and B, and
iii) A screw pair between the links B and C.

Kinematic chains with four lower pairs

The most important kinematic chains are those, which consist of four lower pairs, each pair being either a sliding pair or a turning pair. Many complicated machines are based on combinations of the different inversions of these simple chains.

Quadric chain (Four bar chain)

Quadric chain is any four links kinematic chain. A four bar mechanism is a mechanism having four rigid links with one link fixed. (as shown in fig. 1.31). This chain consists of four turning pairs. The fixed link is referred to as the *frame*. One of the rotating links is called the *driver* or *crank*. The other rotating link is called the *follower* or *rocker* and the floating link is called the *connecting rod* or *coupler*. Depending upon the arrangement and proportions of the links the two pivoted links may both rotate through 360°, one may rotate through 360° while the other oscillates or both may oscillate. Grashof's law provides a very simple test to check whether any of the links in the chain can be a crank.

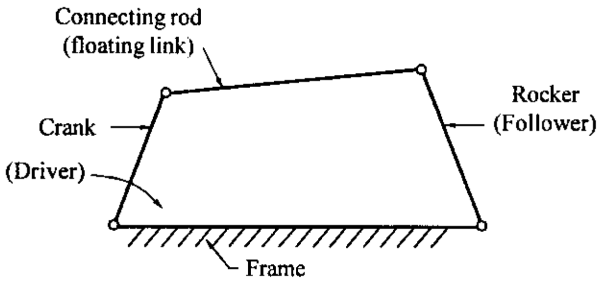


Fig. 1.31

Grashof's criterion: Grashof's theorem states that a four bar mechanism has at least one revolving link if;

$$s + l \leq p + q$$

where s = length of the shortest link

l = length of the longest link

p, q = lengths of other two links

If this equation is not satisfied, the chain is called non-Grashof chain in which none of the links can have complete rotation relative to other links. All three moving links rock. A chain satisfying Grashof's law generates three distinct inversions.

Inversions of four-bar chain (Grashof's chain)

Crank lever mechanism: When any of the two remaining links adjacent to the shortest link is fixed, a crank-lever mechanism results. The shortest link is the crank and is capable of having full rotation (fig. 1.32).

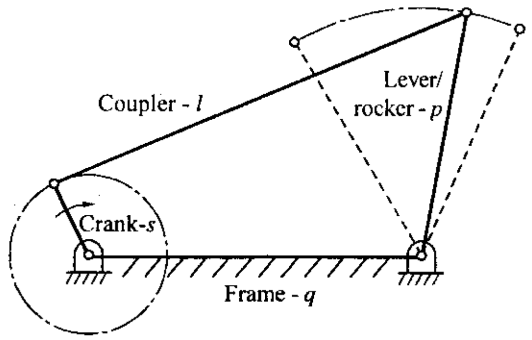


Fig. 1.32 Crank rocker

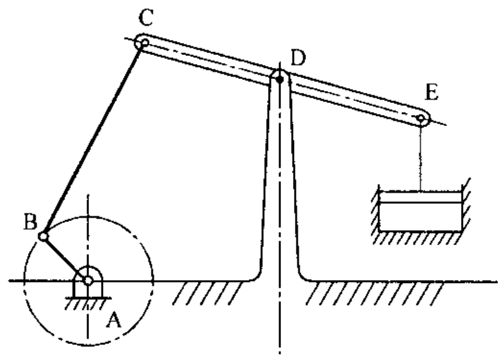
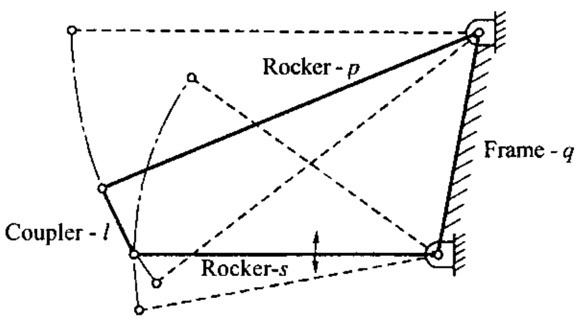


Fig. 1.33 Beam engine

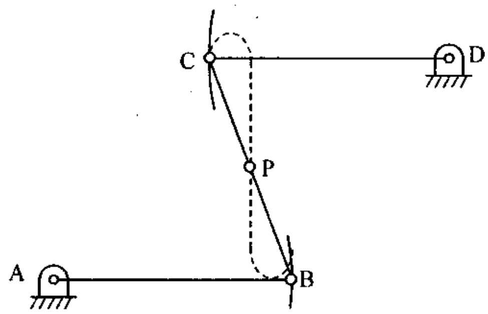
Beam engine: An application of lever-crank mechanism is a beam engine where the crank rotates about the fixed center A and the rocker oscillates about D as shown in fig. 1.33. It is used for converting the rotary motion of the crank into reciprocating motion of the piston. The wiper mechanism used in cars is another application of crank-lever mechanism.

Double lever / rocker mechanism: When the link opposite to the shorter link is fixed, a double lever mechanism results. Note that the two links connected to the frame can only oscillates. However, the coupler is able to complete a full revolution. Pantograph, Ackerman steering gear and Watt's approximate straight line motion mechanisms are the applications of double-lever mechanism.

Watt's straight line mechanism : This is an example of double lever mechanism where both the pivoted links can only oscillate about the fixed center as shown in fig. 1.34b. The links AB and CD oscillate about the fixed centers A and D and the point P on the connecting link CB traces a figure-eight shaped path, a considerable portion of which is approximately a straight line.



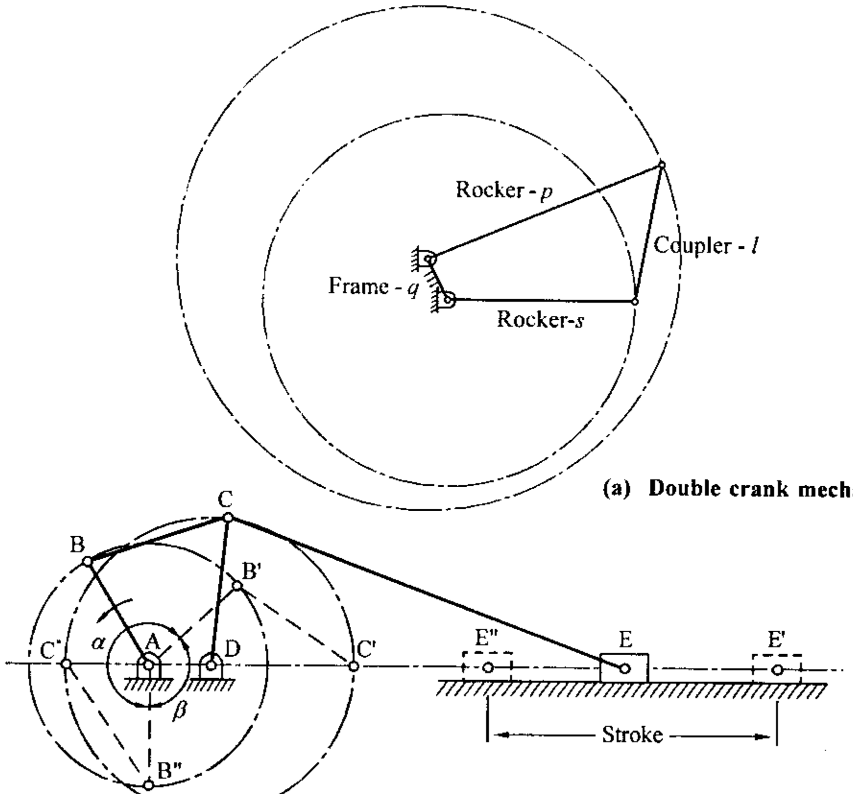
(a) Double rocker



(b) Watt's straight line mechanism

Fig. 1.34

Double crank mechanism: When the shortest link is fixed, a double crank mechanism results. The two links connected to the frame rotates continuously. It is also called a drag-link mechanism. The drag link quick return motion mechanism is the application of double crank mechanism. Quick return motion mechanism is used to reduce the idling time of the machine.



(a) Double crank mechanism

(b) Drag link quick return mechanism

Fig. 1.35

Drag link quick return mechanism : It is a double crank four bar mechanism in which the shortest link is fixed. If the crank AB is driving at a uniform speed, the crank CD will rotate at non uniform speed. The non-uniform rotation of link CD is transformed into quick return reciprocation motion of the ram E by the link CE as shown in fig. 1.35b. When the crank AB rotates through an angle α (arc B' BB") in counter clockwise direction, the link CD rotates through 180° (arc C' C C"). When the crank AB continues to rotate through the angle β (arc B" B'), the link CD again rotates through 180° (arc C" C'). Note that angle α is considerably larger than angle β, i.e., the time required for the first half revolution of the link CD is greater than the time required for the second half revolution. Therefore the movement of the ram from right to left is the cutting stroke. The ratio of α to β is known as time ratio.

Change point mechanism : A change point mechanism is shown in fig. 1.36. It can be positioned such that all the links become collinear. In this mechanism the opposite links are equal in length, i.e., the frame and coupler are the same length and the two pivoted links are the same length. Thus the four links will overlap each other. Coupling rod of a locomotive is an example of this mechanism.

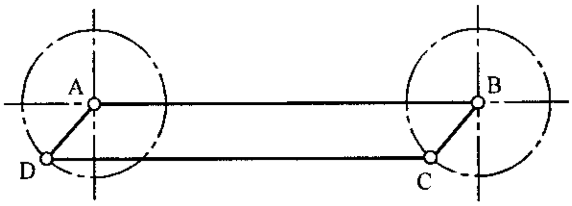


Fig. 1.36

All the four bar mechanism fall into one of the following categories given in table 2.

Table 2

Criteria	*Shortest link*	*Category*
$s + l < p + q$	Frame	Double crank
$s + l < p + q$	Side	Crank rocker
$s + l < p + q$	Coupler	Double rocker
$s + l = p + q$	Any	Change point
$s + l > p + q$	Any	Triple rocker

Single slider crank chain and its inversion

A slider crank is a special case of the four bar mechanism having three turning pairs and one sliding pair. The simplest form in which the slider crank chain appears is that of the reciprocating engine mechanism as shown in fig. 1.37. The slotted link AB is fixed. As the crank BC rotates, the die block D reciprocates. CD is the connecting rod.

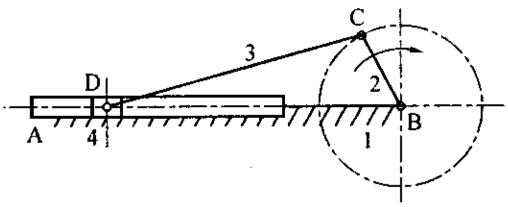


Fig. 1.37

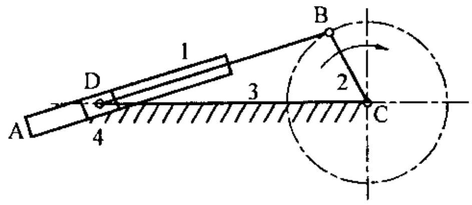


Fig. 1.38

Oscillating cylinder engine : Inversion is obtained by fixing the link CD as shown in fig. 1.38. As the crank BC rotates about C the slotted link AB slides over the die block, which is pivoted to the fixed link at D. The mechanism of oscillating cylinder engine is based upon it. The crank and slotted lever quick return mechanism is another application of this inversion.

Whitworth quick return mechanism : Inversion is obtained by fixing the link BC as shown in fig. 1.39. It is used on slotting and shaping machines. The crank CD rotates at uniform speed. The die block D slides along the slotted link AB and causes this link to revolve about B with a variable speed. The ram R that carries the toolbox is connected to the pin A by the link AR. The ram reciprocates along the line of stroke, which passes through B and is normal to BC. If CD rotates counter clockwise, the time taken to turn from CD_1 to CD_2 will be greater than the time taken to turn from CD_2 to CD_1. Therefore movement of the arm from left to right be the cutting stroke. Quick return motion mechanism is used to reduce the idling time of the machine.

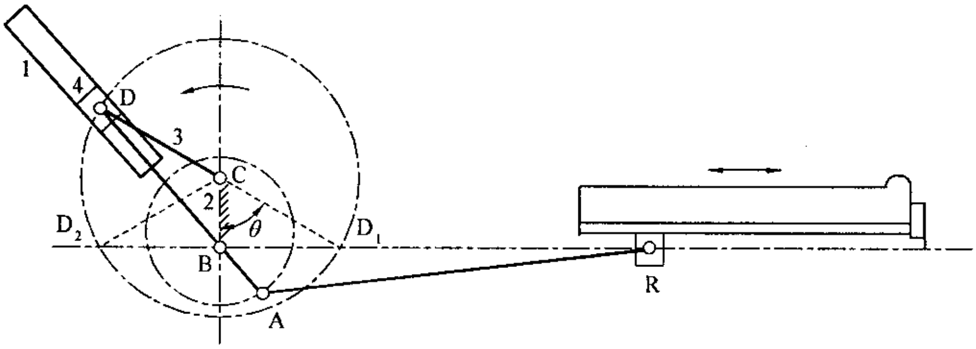


Fig. 1.39

Another application of this inversion is the rotary cylinder engine used in early aircraft engines. The crank is held stationary and the cylinders rotated about the crank.

Pendulum pump or bull engine : The fourth inversion is obtained by fixing the die block D as shown in fig. 1.40(a). As the link BC rotates, the link CD will oscillates about the pin D and the slotted link AB reciprocate along a vertical straight line. The mechanism of pendulum pump or bull engine is based upon it.

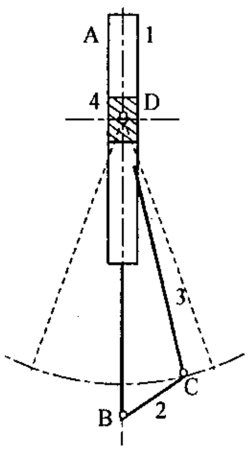


Fig. 1.40(a)

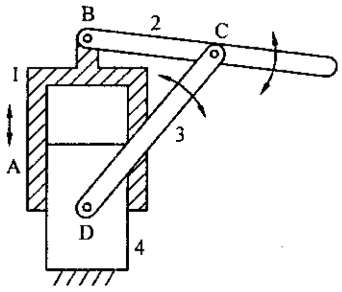


Fig. 1.40(b)

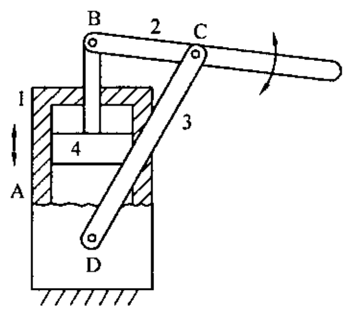


Fig. 1.40(c)

Another application of the inversion is shown in fig. 1.40(b). As the link BC oscillates the link CD will oscillates about pin D and the slotted link AB reciprocate along a vertical straight line. This inversion has certain practical difficulties. To overcome this difficulties, the shapes of piston and cylinder are exchanged as shown in fig. 1.40c. This gives a hand pump mechanism.

Double slider crank chain and its inversion

This kinematic chain consists of two turning and two sliding pairs. Two slide blocks A and B slide along slots in a frame S and the pins A and B on the slide blocks are connected by the link AB as shown in fig. 1.41. Each of the slide block forms a sliding pair with the frame and a turning pair with the link AB. Such a kinematic chain has three inversions.

Elliptical trammel : It is an instrument for drawing ellipses. In the first inversion, the slotted frame S is fixed. Any point, such as P on the link AB except the midpoint of AB will trace an ellipse as the slide blocks A and B slide along their respective slots. From fig. 1.41, the co-ordinate the point P is;

$$x = \text{AP} \cos\theta = a\cos\theta \qquad \text{where} \quad \text{AP} = a$$

$$y = \text{BP} \sin\theta = b\sin\theta \qquad \text{where} \quad \text{BP} = b$$

$$\therefore \qquad \cos\theta = \frac{x}{a} \quad \text{and} \quad \sin\theta = \frac{y}{b}$$

Squaring and adding the above two equations,

$$\sin^2\theta + \cos^2\theta = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

or

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{.... (1)}$$

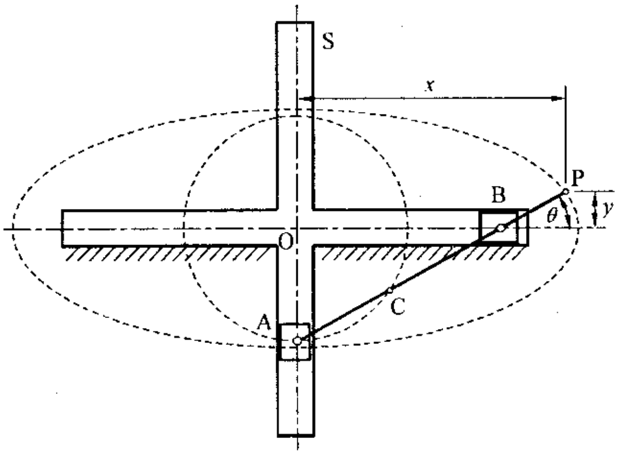


Fig. 1.41

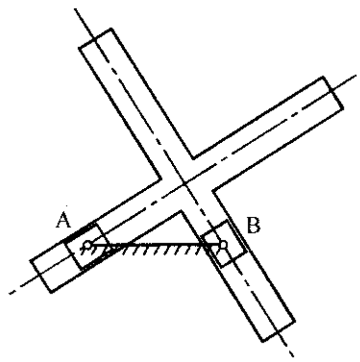


Fig. 1.42

which is the equation of an ellipse with center at O. Therefore, the path traced by the point P is an ellipse having a and b as a semi major and semi minor axis of the ellipse respectively. If the tracing point P is placed at the mid point of AB, then a and b are equal and the equation (1) becomes,

$$x^2 + y^2 = a^2 \qquad \text{..... (2)}$$

which is an equation of a circle of radius a

Another application of this inversion is the donkey pump.

Oldham's coupling: Second inversion is obtained by fixing the connecting link AB as shown in fig. 1.42. For a given angular displacement of any one of the slide block. The frame and other block will turn through the same angular displacement. The Oldham's coupling is an example of this inversion and is shown in fig.1.43.

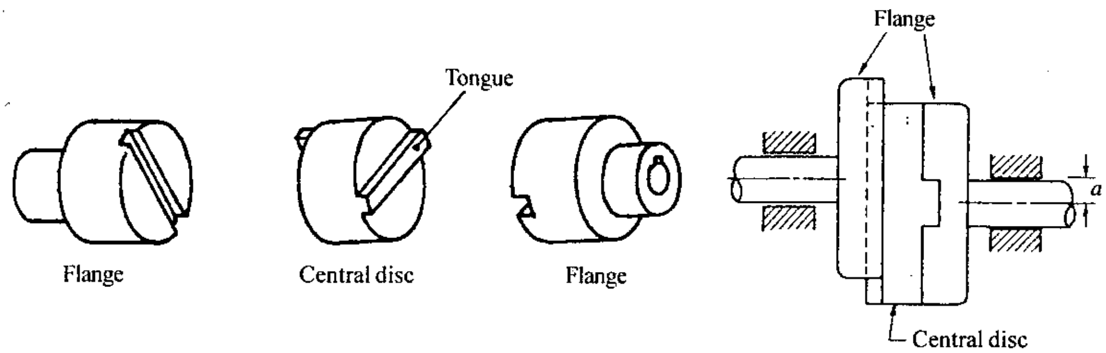


Fig. 1.43

This coupling is used to connect shafts which are parallel but not collinear. It consists of two flanges with a diametric rectangular groove. The central disc contains tongues on either side located at right angles to each other and slides in the grooves of the flanges. When there is a displacement between the axes of the shafts, sliding occurs in both the grooves.

The maximum sliding speed = Distance between the axes of the shaft a × angular velocity of the shaft

Scotch yoke mechanism : Third inversion is obtained by fixing any one of the two blocks A or B as shown in fig. 1.44. The block A is fixed, the link AB can rotate about A as center and thus cause the frame to reciprocate. The fixed block A guides the frame. Scotch yoke mechanism is based on this inversion. The scotch yoke mechanism delivers pure harmonic motion.

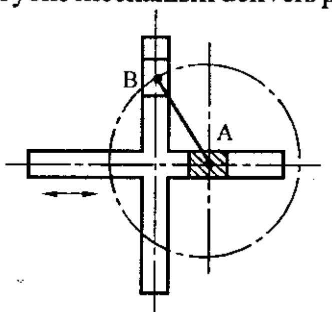


Fig. 1.44

Example 1.16

An Oldham's coupling is used to connect parallel shafts, the distance between their axes being 2 mm. The shafts revolve at a speed of 250 rpm, determine the maximum speed of sliding of the intermediate piece along the groove.

Data :

Distance between the axes of the shaft $a = 2$ mm, Speed of the shaft $n = 250$ rpm

Solution :

Angular velocity of the shaft $\omega = \dfrac{2\pi n}{60} = \dfrac{2\pi \times 250}{60} = 26.18$ rad/s

Maximum sliding speed $= a\,\omega$

$= 2 \times 26.18 = 52.36$ mm/s

REVIEW QUESTIONS

1. Explain the following terms with examples.
   (i) Element, (ii) Kinematic pair, (iii) Mechanism, (iv) Inversion, and (v) Machine.
2. Define a kinematic pair. Explain the various types of kinematic pairs.
3. Give the classification of Kinematic pairs according to type of relative motion between links. Also indicates the degrees of freedom associated with each pair. **(VTU, July 2005)**
4. Distinguish between complete, incomplete, and successful constraint of the relative motion between the two links.
5. Define the following : (i) Lower pair, and (ii) Higher pair
6. Define a kinematic chain and how it differs from a mechanism?
7. Differentiate between : (i) Mechanism and machine, (ii) Machine and structure.
8. Distinguish between (i) Complete constraint and successful constraint, (ii) Open pair and closed pair, (iii) Mechanism and machine, (iv) Structure and kinematic chain.
9. Differentiate between, (i) Lower pair and higher pair (ii) Constrained motion and unconstrained motion, (iii) Machine mechanism
10. Define mobility of a mechanisms and write the Grubler's mobility equation for planar mechanism.
11. Explain Grublers criterion for determining the degree of freedom for mechanism. **(VTU, July 2006)**

12. Determine the mobility of the following devices shown in fig. 1.45. **[Ans.** 2, 1, 0]

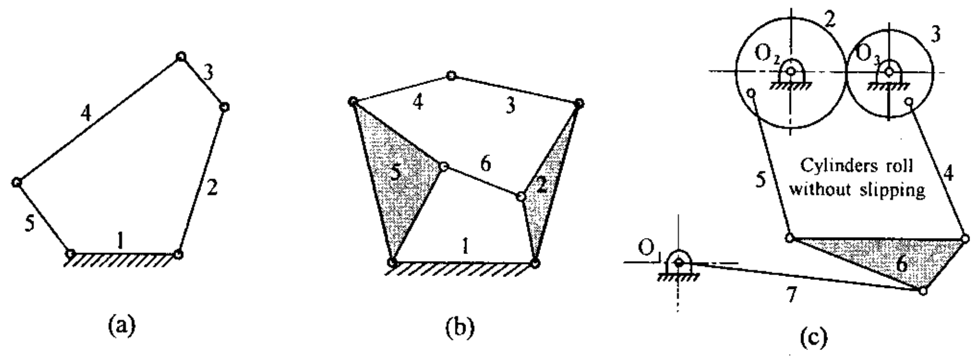


Fig. 1.45

13. Write notes on kinematic chain with three lower pairs.
14. Sketch and explain the following :
    (i) Four bar chain and its inversion, (ii) Single slider crank chain and its inversion, (iii) Double slider crank chain and its inversion.
15. State and explain Grashof's criterion
16. What is an inversion? Explain any three inversions of a four bar chain.
17. What are inversions of a mechanism? Sketch the inversions of slider crank chain and name the mechanism obtained.
18. Describe the various inversions of a single slider crank chain with sketches.
19. Explain the inversions of double slider crank chain with the aid of neat sketches. **(VTU, Jan 2006)**
20. Explain the working of ellipse trammel and show how it is useful for drawing an ellipse.
21. Sketch and explain the working of elliptical trammel. Prove that it traces an ellipse.
22. Describe the construction of Oldham's coupling and state for what purpose it is used?
23. Describe the following quick-return mechanisms with suitable sketches.
    (i) Drag link, (ii) Whitworth, and (iii) crank and slotted lever mechanism.
24. Distinguish between: i) Complete constraint and successful constraint ii) Open pair and closed pair, iii) Structure and kinematic chain iv) Mechanism and machine. **(VTU, Jan 2008)**
25. Explain with neat sketches, three inversions of double slider crank chain. **(VTU, Jan 2008)**

2

MECHANISMS WITH LOWER PAIRS

Lower pair has a degree of freedom as one, as each point on one element of the pair can move only along a line or curve relative to the other element. Since lower pairs involve surface contact, it follows that lower pairs can be more heavily loaded for the same unit pressure. They are considerably more wear resistant. The examples of lower pairs are: turning pairs, prismatic pairs and screw pairs.

The mechanisms discussed in this chapter meet a variety of common needs in mechanical engineering practice. The quick-return mechanisms, automotive steering mechanisms and generation of a straight line by a simple linkage mechanism are the most common linkage mechanism in practical use. Likewise, linkages that can reproduce the path traced by one point at another tracing point with a change in scale find many uses ranging from machine tools to robotic mechanisms. Yet another recurring need in practical linkage design is for indexing. Intermittent, timed advancement of a drive in a constant direction. This mechanism is mainly used in packaging machinery.

Quick - return mechanisms

In many applications, mechanisms are used to perform repetitive operations. In these repetitive operations there is usually a part of the cycle when the mechanism is underload, called *working stroke*. The remaining part of the cycle called the *return stroke*. The mechanism simply returns to repeat the operation without load. The ratio of the time for working stroke to the time for the return stroke is known as *time ratio*. To produce quick return, the time ratio must be greater than unity and as large as possible. Quick return mechanisms are used on machine tools to give a slow cutting stroke and a quick return stroke for a constant angular velocity of the driving crank. The most commonly used types of quick return mechanisms are: drag link, Whitworth, crank-shaper, and offset slider crank mechanism.

Drag link mechanism : It is a double crank four bar mechanism in which the shortest link is fixed. If the crank AB is driving at a uniform speed, the crank CD will rotate at non uniform speed. The non-uniform rotation of link CD is transformed into quick return reciprocation motion of the ram E by the link CE as shown in fig. 2.1. When the crank AB rotates through an angle α (arc B' BB") in counter clockwise direction, the link CD rotates through 180° (arc C' C C"). When the crank AB continues to rotate through the angle β (arc B" B'), the link CD again rotates through 180° (arc C" C'). Note that angle α is considerably larger than angle β, i.e., the time required for the first half revolution of the link CD is greater than the time required for the second half revolution. Therefore the movement of the ram from right to left is the cutting stroke. The ratio of α to β is known as time ratio.

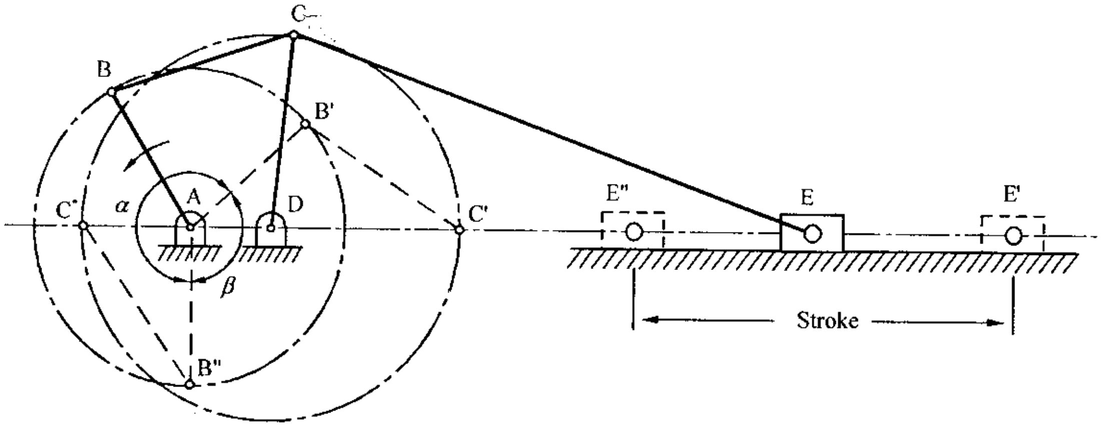


Fig. 2.1

Whitworth quick return mechanism : A Whitworth quick-return mechanism is shown in fig. 2.2. It is used on slotting and shaping machines. The crank CD rotates at uniform speed. The die block D slides along the slotted link AB and causes this link to revolve about B with a variable speed. The arm R that carries the toolbox is connected to the pin A by the link AR. The arm reciprocates along the line of stroke, which passes through B and is normal to BC. If CD rotates counter clockwise, the time taken to turn from CD_1 to CD_2 will be greater than the time taken to turn from CD_2 to CD_1. Therefore movement of the arm from left to right be the cutting stroke. Quick return motion mechanism is used to reduce the idling time of the machine.

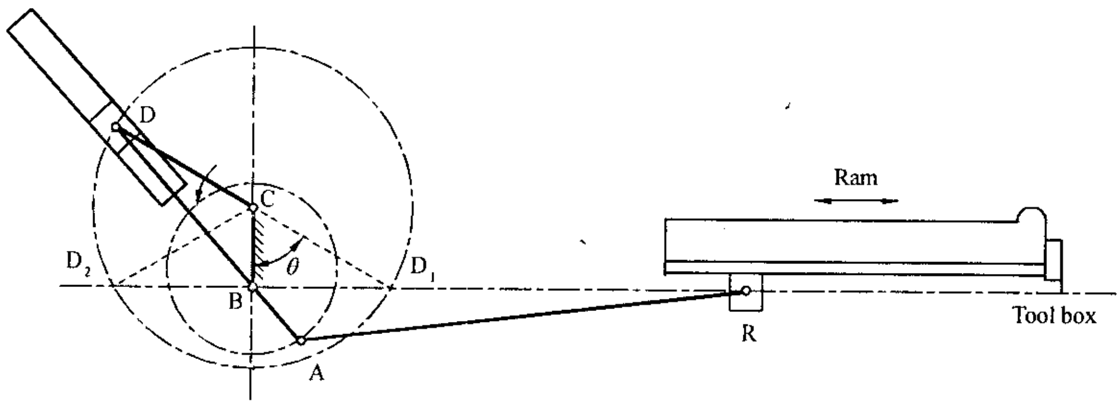


Fig. 2.2

Crank and slotted lever mechanism : This mechanism employs an inversion of the slider crank chain, and is illustrated in fig. 2.3. As the crank OA rotates about the fixed center O, the die block A moves along the link BC and causes the link BC to oscillates about the fixed axis C. The oscillation of the link BC is transformed into quick return reciprocation motion of the ram D by the link BD. If the driving crank OA rotates with uniform angular speed in the counter clockwise direction, the ram D will have a slow working stroke to the left and quick return stroke to the right. The time ration is α/β. This mechanism is widely used in shaping machines, slotting machines and in rotary engines.

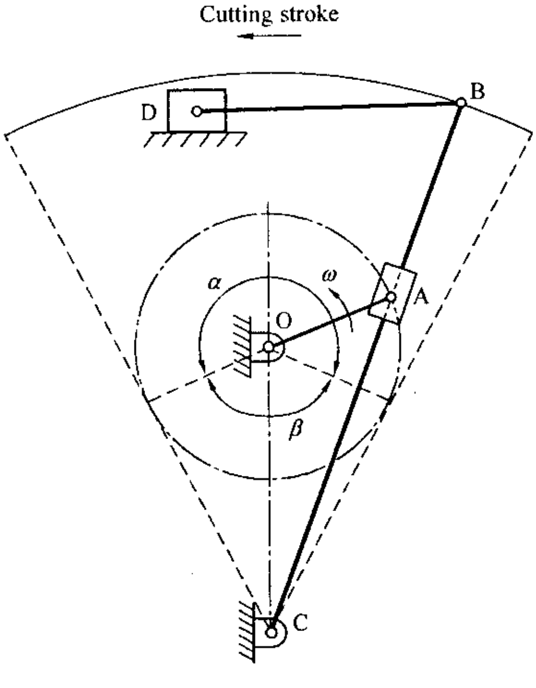


Fig. 2.3

Straight-line generators

An obvious way to have constrained motion of a point in a mechanism along a straight line is to use a sliding pair. Sliding pairs, however, have a limitation in that they are bulky and are susceptible to rapid wear. Therefore it is necessary to obtain straight line motion using turning pairs only. The straight line generator may either use all turning pairs of combination of turning pairs with one sliding pair.

There are two types of straight line motion mechanisms.

1. Exact straight line motion mechanism and
2. Approximate straight line motion mechanism.

Compared to exact straight line motion mechanism, approximate straight line mechanisms have fewer links. Most of the exact straight line generators uses mechanisms with six or more links. Coupler curves are used to generate approximate straight line between two selected tolerance limits on linearity.

Exact straight line motion mechanisms

Condition for generating exact straight line motion: Let a line AP turn about A as center and let the position of the point P be such that the product AQ × AP is constant. Then the path of P will be a straight line perpendicular to the diameter AB of the circle along the circumference of which Q moves.

From Fig. 2.4 the triangles AQB and APX are similar.

Therefore $$\frac{AQ}{AB} = \frac{AX}{AP}$$

or $$AX = \frac{AQ \times AP}{AB}$$

Since AB is constant as it is the diameter of the circle so that if the product AQ × AP is constant, AX will also be constant. Hence the point P moves along with the straight line PX that is perpendicular to AB. Number of mechanisms have been devised to satisfy the above condition. Two of them are Peaucellier and Hart mechanism.

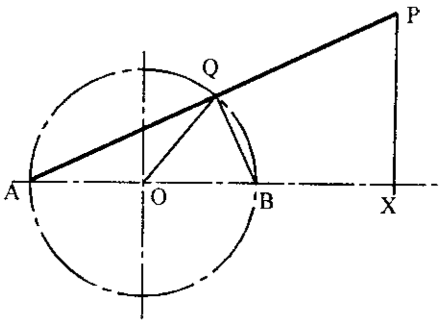


Fig. 2.4

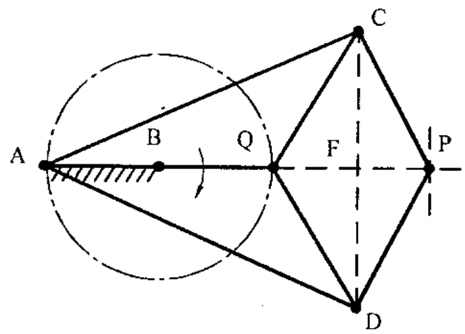


Fig. 2.5

Peaucellier mechanism : It gives exact straight line motion and is an eight-link mechanism. The link AB is fixed as shown in fig. 2.5.

Length AD = AC
Length AB = BQ and
Length QC = CP = PD = DQ

The tracing point P will describe a straight line perpendicular to AB as the link BQ rotates about B.

Proof : The diagonals of a rhombus QCPD bisect at right angles at F. Consider the right angled triangles ACF and CPF,

$$AC^2 = AF^2 + FC^2 \qquad \text{..... (1)}$$

and

$$CP^2 = FC^2 + FP^2 \qquad \text{..... (2)}$$

Subtracting (2) from (1)

$$AC^2 - CP^2 = AF^2 + FC^2 - FC^2 - FP^2 = AF^2 - FP^2$$

$$= (AF - FP)(AF + FP) = AQ \times AP$$

Since AC and CP are constant (length of the links) so that the product AQ × AP is a constant and therefore P traces a straight path normal to AB.

Another interesting property is that if AB is not equal to BQ, point P can be made to trace a circular arc of very large radius.

Approximate straight line motion mechanisms

The following are the approximate straight line motion mechanisms. 1) Watt mechanism, 2) Grasshopper mechanism, 3) Robert's mechanism, and 4) Tchebicheff's mechanism.

Robert mechanism : Fig 2.6 shows Robert's mechanism, which produces approximately straight line. This is a four bar chain in which the length AB = BP = PC = CD and AD = 2 BC. The tracing point P moves very nearly along the line AD. The accuracy of the motion can be increased by increasing the ratio of height of the mechanism to its width.

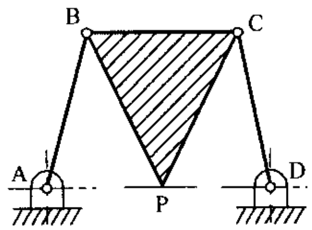


Fig. 2.6

Pantograph

A pantograph is a copying mechanism. It is used to reproduce path described by a point either to an enlarged scale or a reduced scale. Links 2, 3, 4 and 5 are pin jointed at A, B, C and D with opposite links equal in length to form a parallelogram as shown in fig. 2.7. The link 3 is extended and containing point P. Q is the point of intersection of the line AP and the link CD. The point P and Q both lie on a straight line, which passes through the fixed pin A. It can be shown that in these circumstances the point P will reproduce the motion of the point Q to an enlarged scale or alternatively the point Q will reproduce the motion of point P to a reduced scale. In order for the motion of Q to be parallel to that of

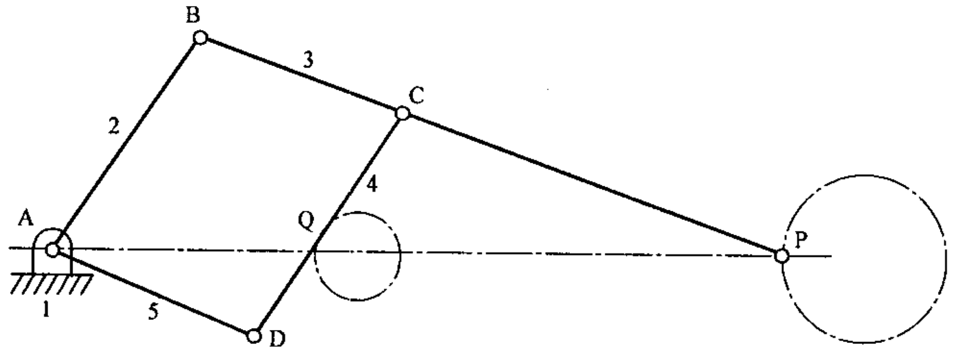


Fig. 2.7

P for all positions it is necessary that the ratio AP/AQ be constant. For all positions of P, triangles ADQ and PCQ are similar, since their three sides are always parallel.

Hence

$$\frac{AQ}{AD} = \frac{PQ}{PC} \quad \text{or} \quad \frac{AQ}{PQ} = \frac{AD}{PC}$$

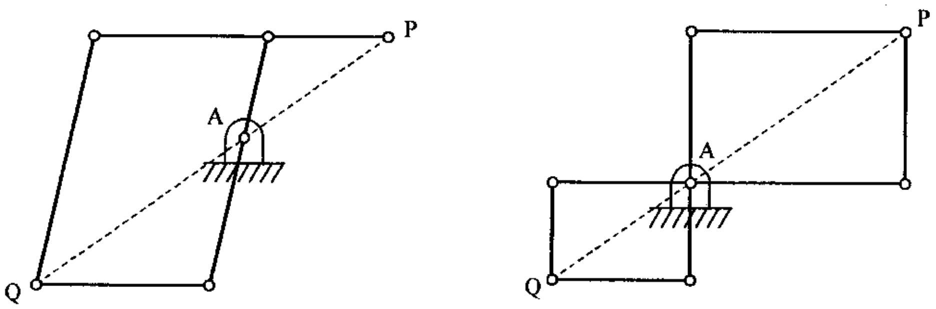
Since the length of the links AD and PC are constant,

$$\therefore \qquad \frac{AQ}{PQ} = \text{constant}$$

Therefore the displacement of Q is parallel to displacement of P and

$$\frac{\text{Size of the figure at P}}{\text{Size of the figure at Q}} = \frac{AP}{AQ}$$

The other forms of pantographs are shown in fig. 2.8.



Second variation of Pantograph Third variation of Pantograph

Fig.2.8

Uses : Pantograph is used to reduce or enlarge drawings and maps. It is also used for guiding cutting tools or cutting torches to duplicate complicated shapes. Pantograph is widely used in engraving or profiling machines. It is used as an indicator rig in order to reproduce to a small scale the displacement of the cross head of reciprocating engine, which gives the displacement of engine piston.

Toggle mechanism

In slider crank mechanism, as the crank approaches one of its dead-center position, the movement of the slider approaches zero. The ratio of the crank movement to the slider movement approaches infinity, which is proportional to the mechanical advantage. This principle

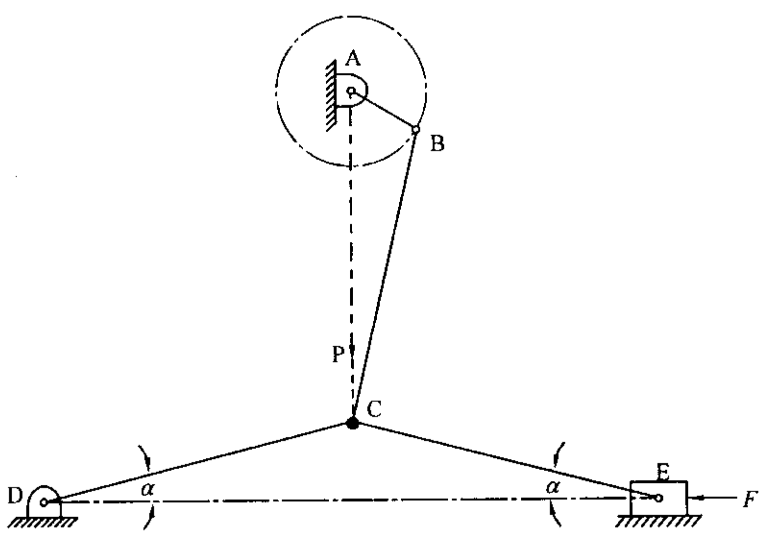


Fig. 2.9

is used in toggle mechanism. A toggle mechanism is used where a large force acting through a short distance is required. Fig. 2.9 shows a toggle mechanism. Links CD and CE are of same length. Resolving the forces at C vertically, we get

$$\frac{F}{\cos\alpha}\sin\alpha = \frac{P}{2} \quad \text{or} \quad F = \frac{P}{2\tan\alpha}$$

Thus for a given value of P, as the links CD and CE approaches collinear position ($\alpha \to 0$), the force F rises rapidly. Toggle mechanisms are used in toggle clamps, riveting machines, punch presses, stone crushers, and other applications where a large force results from a small applied force. The toggle principle is also effectively used in switches, circuit breakers, and other mechanisms where a snap action is required.

Automobile steering gear

It is a mechanism for changing the direction of motion of two or more wheel axles with respect to chassis, in order to move the car in any desired path. The front wheels are mounted on short separate axles which are pivoted to the chassis of the vehicle. The rear wheels have a common axle and this is fixed. Steering is usually effected by turning the axes of rotation of the two front wheels relative to the chassis of the vehicle. If the car is making a right turn as in fig. 2.10, the axis of the right wheel must swing about pin C through a greater angle than the left wheel about A.

The condition for good steering is that the relative motion between the wheels of the vehicle and the road surface must be one of pure rolling. This is possible when the wheels move along concentric arcs of circle about an instantaneous center of rotation. To satisfy the above condition, the swing of the two axes would be such that their center lines extended would always intersect on the center line of the rear axle at G. Then all parts of the car would be moving about a vertical axis through G and the tendency of the wheels to skid would be reduced to a minimum.

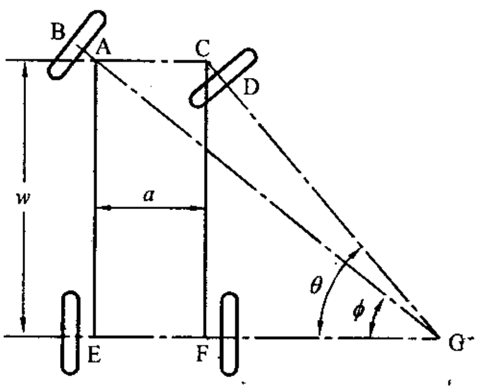


Fig. 2.10

Let w = wheel base = AE = CF

and a = wheel track = AC = EF

from the fig. 2.10

$$AC = EF = EG - FG \qquad \text{..... (1)}$$

From triangle AEG

$$\tan\phi = \frac{AE}{EG}$$

$$EG = \frac{AE}{\tan\phi} = AE\cot\phi$$

Similarly from triangle CFG,

$$FG = CF\cot\theta$$

Substituting the value of EG and FG in equation (1) we get,

$$AC = AE\cot\phi - CF\cot\theta$$

$$\therefore \quad a = w\cot\phi - w\cot\theta = w(\cot\phi - \cot\theta)$$

or $$\cot\phi - \cot\theta = \frac{a}{w} \qquad \text{..... (2)}$$

Ackerman steering gear

It consists only of the turning pairs and is based on a four bar chain in which the two longer links AC and PQ are unequal in length while the two shorter links AP and CQ are equal in length as in fig. 2.11. In the mid position when the car is moving along a straight path, the links AC and PQ are parallel and the links AP and CQ each inclined at the angle α to the longitudinal axis of the car. The value of ϕ obtained for a given value of θ would depend upon the ratio AP/AC and the angle α. This gear gives correct steering for three positions only.

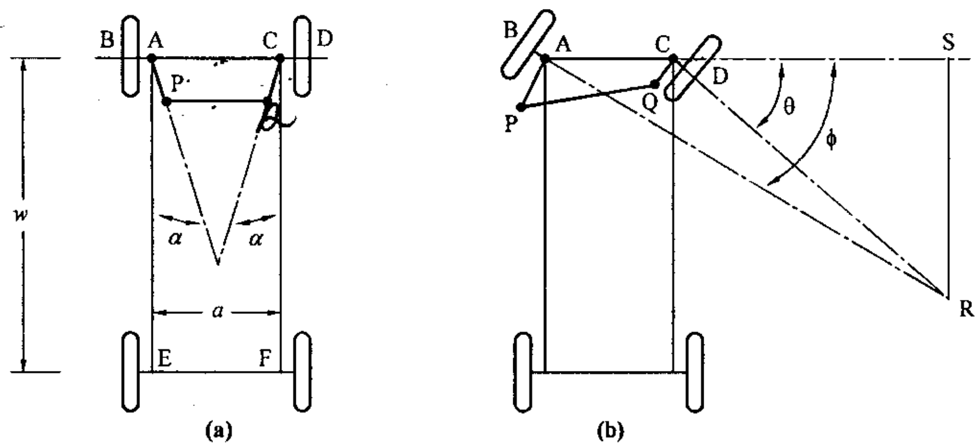


Fig. 2.11

1. When the car is moving along a straight path ($\theta = 0°$).
2. When moving at one correct angle to the right depending upon AP/AC ratio and the angle α ($\theta \simeq 24°$).
3. Similar position when the car is moving to the left ($\theta \simeq -24°$).

In all other positions, pure rolling is not obtainable.

For other values of angle θ, the wheels tend to follow the path along circular arcs that do not have a common center. The instantaneous center does not lie on the axis of the rear wheels but lies on a line parallel to the rear axle and move towards the front as shown in fig. 2.11*b*.

$$\therefore \quad \cot \phi - \cot \theta = \frac{AC}{SR} = \frac{a}{SR}$$

Ackerman steering is still the most commonly used steering gear. The advantage of the Ackerman steering gear lies in the use of revolute joint rather than the sliding pairs.

Intermittent motion mechanism

An intermittent motion mechanism is a linkage which converts continuous motion into intermittent motion.

Ratchet mechanism : This mechanism is used to produce intermittent circular motion from an oscillating or reciprocating member. A ratchet consists of a ratchet wheel 2 and a pawl 3 as shown in fig. 2.12. The pawl is held against the wheel by gravity or by spring. As the pawl lever 4 is raised, the pawl drives the wheel counter clockwise. As the pawl lever is lowered, the pawl slides over the ratchet teeth. A second pawl 5 is used to prevent the ratchet wheel from reversing. Ratchets are used in feed mechanisms, lifting jacks, clocks, watches, and counting devices.

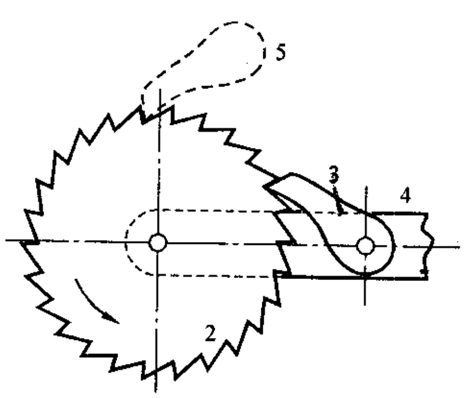


Fig. 2.12

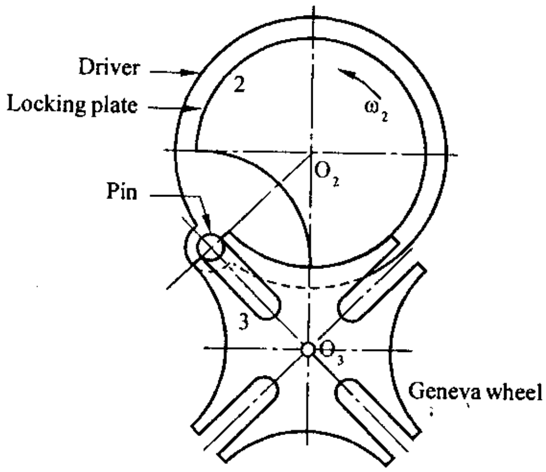


Fig. 2.13

Geneva wheel indexer : In this mechanism plate 2 is the driver and contains a pin which engages slots in the driven link 3. The slots are positioned so that the pin enters and leaves them tangentially. Thus an advantage of this mechanism is that it provides indexing without impact loading. Three or more slots may be used in the Geneva wheel. In the particular mechanism shown in fig. 2.13, four slots are used in the wheel. The driven member rotates one-fourth of a revolution for every revolution of the driver. The locking plate, which is mounted on the driver, prevents the driven member from rotating except during the indexing period. This mechanism has long been so used to prevent over winding of main springs in clocks and watches, and feeding the strip of film in a quick advance in the early motion-picture projectors.

REVIEW QUESTIONS

1. What do you understand by the term quick-return motion mechanism and why it is necessary?
2. Sketch and explain drag link quick-return motion mechanism.
3. Sketch and explain whitworth quick return motion mechanisms.
4. Sketch and explain the crank and slotted lever quick return motion mechanism.
5. What are straight line motion mechanisms? How are they classified?

6. Derive the condition for a mechanism to trace an exact straight line path. Sketch Peaucellier's mechanism and show that this mechanism satisfing the above condition.
7. Explain the construction and working of Peaucellier mechanism by means of a sketch. Prove that it generates an exact straight line. **(VTU, July 2006**
8. Describe Roberts approximate straight line motion mechanism with a suitable sketch.
9. What is a pantograph and what are its uses? Give a neat sketch of a pantograph and explain its working principles.
10. Explain a mechanism which is used to enlarge a given figure with proof.
11. Describe toggle mechanism. What are its uses?
12. State and prove the condition that must be satisfied by the steering mechanism of a car in order that the wheels may have pure rolling motion when rounding a curve?
13. Write short notes on Ackermann steering gear.
14. Obtain the condition for correct steering for a four wheeled vehicle. Sketch and explain the working of Ackermann steering gear. State its merits and demerits. **(VTU, July 2006)**
15. What is intermittent motion mechanism? Explain the following intermittent motion mechanism: (i) Geneva, and (ii) Ratchet.
16. Sketch and explain the following mechanisms: Geneva wheel, Ratchet and pawl mechanism, Ackermann steering mechanism. **(VTU, Jan. 2007)**
17. With neat proportionate diagrams explain the principle and working of following mechanisms. (a) Whitworth mechanism, (b) Geneva mechanism, (c) Toggle mechanism. **(VTU, Jan 2008)**